Using High Performance Computing for the Application of PINNs in Cardiovascular Models

Elena Gómez del Pozo[1], Karolina Tlałka[1,2], Karol Zając[1], Maciej Malawski[1,3]

[1] Sano Centre for Computational Medicine, Krakow, Poland;
[2] University of Sheffield, Sheffield, United Kingdom; [3] AGH University of Krakow, Krakow, Poland.

Problem: Existing complex cardiovascular models have multiple parameters, which complicates the task of personalization [1].
Solution: Infusing complexity to the models through Physically Informed Neural Networks (PINNs), keeping parameter pool small yet still capturing complex spikes that are relevant for clinical assessments. [2,3]

MATERIALS AND METHODS

• Two-Chamber Cardiovascular Model

We used a single-circuit lumped-parameter model with two heart chambers: left atrium and ventricle. Pulmonary circulation and right heart excluded.

Governing Ordinary Differential Equation (ODE) system:

$$d_1 : \frac{dp_{lv}}{dt} = (Q_{mv} - Q_{av})E_v + \frac{P_{lv}}{E_v}\frac{dE_v}{dt}$$

$$d_2 : \frac{dp_{la}}{dt} = (Q_{sv} - Q_{mv})E_a + \frac{P_{la}}{E_a}\frac{dE_a}{dt}$$

$$d_3 : \frac{dp_{sa}}{dt} = \frac{Q_{av} - Q_s}{C_{sa}}$$

$$d_4 : \frac{dp_{sv}}{dt} = \frac{Q_s - Q_{sv}}{C_{sv}}$$

$$d_5 : \frac{dV_{lv}}{dt} = Q_{mv} - Q_{av}$$

$$d_6 : \frac{dV_{la}}{dt} = Q_{sv} - Q_{mv}$$

$$d_7 : \frac{dQ_{av}}{dt} = \begin{cases} \frac{dp_{lv}/dt - dp_{sa}/dt}{Z_{ao}}, & \text{if } P_{LV} - P_{sa} > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$d_8 : \frac{dQ_{mv}}{dt} = \begin{cases} \frac{dp_{la}/dt - dp_{lv}/dt}{R_{mv}}, & \text{if } P_{LA} - P_{LV} > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$d_9 : \frac{dQ_s}{dt} = \frac{dp_{sa}/dt - dp_{sv}/dt}{R_{sv}}$$

$$d_{10} : \frac{dQ_{sv}}{dt} = \frac{dp_{sv}/dt - dp_{la}/dt}{R_{sv}}$$

• PINN Architecture

A neural network introduces corrections to the governing ODE system [4]. The corrected equations are integrated over the cardiac cycle for a given set of parameters and initial conditions (ICs), producing predicted trajectories $\mathbf{M}_{pred}$, which are compared against the 4-chamber reference $\mathbf{M}_{target}$ [5].

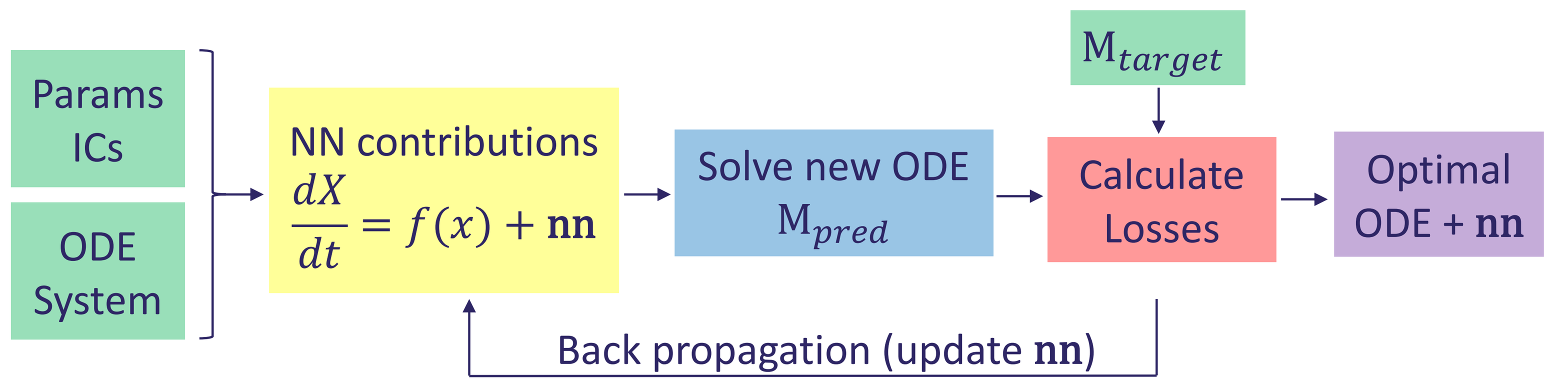

Figure 1. PINN training diagram. Green: inputs, purple: outputs.

• Loss function design

The following loss terms were selected to guide the network toward 4-chamber dynamics, balancing solution accuracy with physical consistency.

Loss term	Expression
Data (D)	$L = M_{pred} - M_{target}$
First derivative (FD)	$L = \mathrm{M}'_{pred} - \mathrm{M}'_{target}$
Physics (Ph)	$L = ODE(\mathrm{M}_{pred})$
Mass conservation (M)	$L = \int_0^{\tau} (Q - \overline{Q})\, dt$
Zero-mean (ZM)	$L = \sum \mathrm{nn}_i \text{ for } i: \{\text{all variables}\}$
Negativity (N)	$L = V_i \text{ if } V_i < 0 \text{ for } i = \mathrm{LV, LA}$
Periodicity (T)	$L = \sum X_0 - X_\tau \text{ for } X: \{\text{all variables}\}$

Table 1. Loss functions evaluated in this study.

RESULTS

A systematic loss study was conducted evaluating the next loss configurations applied globally across all six state variables and individually per variable. The next is a heat map of how loss fluctuated for each training scenario.

	D	D+FD	D+Ph	D+M	D+ZM	D+N	D+T	FD	FD+Ph	FD+M	FD+ZM	FD+N	FD+T
P_{LV}	3.8143	2.4888	3.1517	2.688	1.5298	3.152	3.895	-0.56244	-0.52782	-0.91614	-0.02427	-0.56244	-0.55479
P_{LA}	0.6843	0.6341	0.6115	0.9307	0.202	0.6067	0.6691	0.06963	0.00014	0.4069	0.00022	0.06963	0.00026
P_{SA}	-0.187	-0.038	-0.164	-0.27	0.012	-0.205	-0.162	1.41723	-0.03514	1.46286	3E-05	1.41723	0.0017
P_{SV}	-4.215	-4.476	-4.476	-4.458	-3.869	-4.476	-4.477	0.00072	0.00087	0.01799	0.0007	0.00072	0.00066
V_{LV}	0.0249	0.0204	0.025	0.0252	0.0249	0.0249	0.0458	6E-05	7E-05	0.00044	0.0001	6E-05	0.00012
V_{LA}	-0.037	-0.05	-0.037	-0.036	0.013	-0.037	-0.085	7E-05	8E-05	0.00067	7E-05	7E-05	2E-05
All	-1.892	-2.64	-2.437	-2.875	-3.794	-2.454	-2.229	0.45373	-0.03061	-0.16938	-0.02424	0.45373	-0.31253

Loss increase ← → Loss decrease

Figure 2. Training loss variation ($\Delta L = L_f - L_i$) for several configurations.

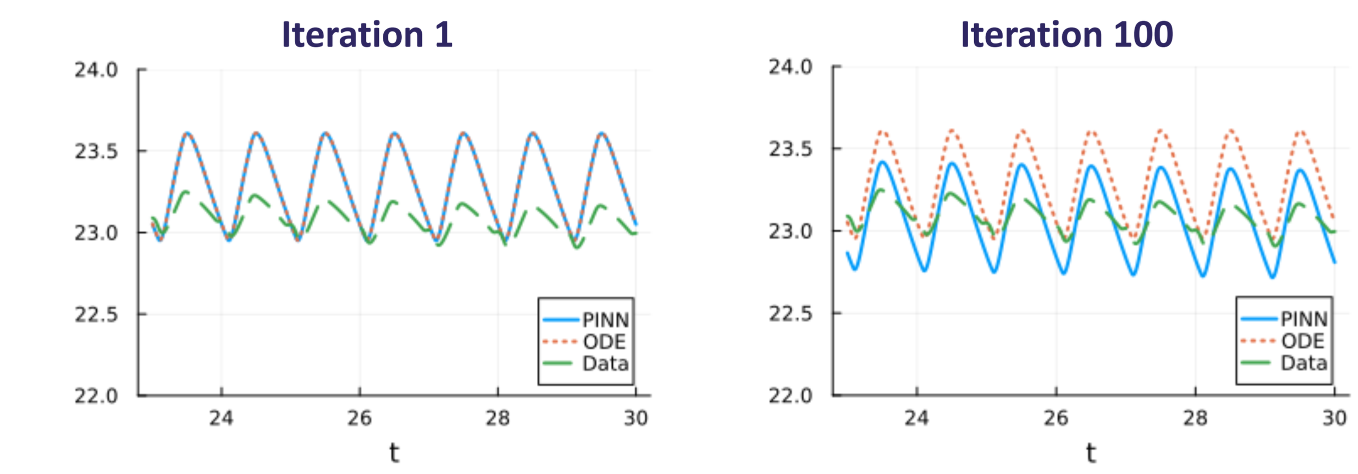

Figure 3. Best training (D+T: Data and periodicity loss on P_{SV}) iterations 1 and 100.

CONCLUSIONS

- A PINN framework was developed to enrich a 2-chamber ODE cardiovascular model toward 4-chamber hemodynamic complexity.
- Loss function selection proved critical, with strong variable-dependent sensitivity observed across the six state variables and seven loss configurations.
- Data loss consistently degraded training on variable #1 (PLV) while improving it for variable #4 (PSV).
- Zero mean loss consistently attenuates the gains of other losses when applied per variable yet has been proved beneficial in the all-variable configuration.

FUTURE WORK

- Extension to multi-patient training: training on different ICs and parameter sets, treating them as a virtual cohort, with each case paired to its closest 4-chamber analogue (via M_{target}).
- Sensitivity and identifiability analysis: Assess how sensitive the learned dynamics are to variations in parameters and IC and identify the most critical parameters for an accurate reconstruction.
- Generalizability assessment: test model performance and robustness across the virtual cohort to quantify its ability to generalize to unseen parameter regimes and ICs.

REFERENCES

[1] H. Saxton, Ph.D. thesis, Sheffield Hallam University, 2025.
[2] M. Raissi, P. Perdikaris, and G. E. Karniadakis, DOI:10.1016/j.jcp.2018.10.045.
[3] Ahmad et al., DOI: 10.1016/j.array.2026.100688
[4] W. Perczak and M. Dydek. PINNFuser.jl https://github.com/mdydek/PINNFuser.jl
[5] Y. Shi P. Lawford, and R. Hose, DOI: 10.1186/1475-925X-10-33

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 857533 and from the International Research Agendas Programme of the Foundation for Polish Science No MAB PLUS/2019/13.

Ministry of Science and Higher Education Republic of Poland

European Funds Smart Growth
Republic of Poland
FNP Foundation for Polish Science
European Union European Regional Development Fund

Funded by the initiative of the Minister of Science and Higher Education "Support for the activities of Centers of Excellence established under the Horizon 2020 program"

We gratefully acknowledge Polish High-Performance Computing infrastructure PLGrid (HPC Centre: ACK Cyfronet AGH) within computational grant no. PLG/2026/019400.